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by Ray H. Wright and Raymond L. Barger

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WIND-TUNNEL LIFT INTERFERENCE ON SWEPTBACK WINGS IN RECTANGULAR TEST SECTIONS WITH SLOTTED TOP AND BOTTOM WALLS

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SUMMARY

A theory is presented for the boundary-induced upwash interference on a sweptback lifting wing mounted at the center of a rectangular wind-tunnel test section with slotted top and bottom walls and closed side walls. A sample calculation for a wing spanning 0.7 of the width of a square tunnel shows the spanwise variation in the interference characteristic of this type of test section. For this example, the wing sweep did not have a large effect on the interference.

INTRODUCTION

Because of the reliance placed on wind-tunnel tests in the design of aircraft and in the prediction of their performance, it is important to know how the test results are affected by the interference of the wind-tunnel boundaries, and, if the interference is significant, to be able to make at least first-order corrections. One type of interference results from the interaction of the tunnel boundaries on the lift produced by a wing. If the lift is directed upward, the interaction of closed tunnel boundaries produces a relative upwash, whereas the interaction of open boundaries produces a relative downwash. If the tunnel boundary is partly open and partly closed (e.g., slotted), the interference may produce either upwash or downwash depending on the particular wall configuration.

The wind-tunnel boundary interference due to lift on wings mounted at the centers of rectangular test sections with slotted top and bottom walls and solid side walls can be calculated by use of theoretical developments presented in references 1 and 2. Reference 1 gives the interference at the center of the tunnel and reference 2 gives an average interference over the wing span of the model. However, if the span of the wing is not small relative to the width of the test section, the variation of the interference over the span may be significant. If, in addition, the wing is swept back, the sweep may affect the interference. The tunnel boundary interference due to lift of sweptback wings in wind tunnels with open or closed boundaries is treated in reference 3, but no comparable treatment for slotted boundaries has previously been available.

In the investigation reported herein a theory for the boundary-induced upwash on a sweptback lifting wing centrally located in a test section with slotted top and bottom walls and closed side walls has been developed. Application has been made to a sweptback wing spanning 0.7 of the width of a square test section with slotted top and bottom walls and closed side walls.

SYMBOLS

A	area; also area on which C_L is based
$A(\omega,g)$	parameter in solution of transformed Laplace equation
a	segment of airfoil span included in a discrete point representation
b	width of test section
C	cross-sectional area of test section
C_L	lift coefficient of model
d	distance between centers of two adjacent slots
e	base of Napierian logarithm
g	variable of transformation on $(y - y_1)$
G	exponential Fourier transform of φ on $(x - x_1)$ and $(y - y_1)$
G_d'	exponential Fourier transform of Ω_d' on $(y - y_1)$
G_d	limit of G_d' as $\alpha \rightarrow 0$
$G_1(\omega,g,z)$	antisymmetric solution of transformed Laplace equation
h	height of test section
$i = \sqrt{-1}$	

j	summation index
K_1	modified Bessel function of the second kind
k	any odd integer, index of summation
l	restriction constant of slotted walls (developed in ref. 2)
L	total lift of model
ΔL	element of lift
M	Mach number
n	any even integer, index of summation
$p = h\omega$	
P_i,P_j	points $\left(\frac{x_i}{h},\frac{y_i}{h},0\right)$ and $\left(\frac{\xi_j}{h},\frac{\eta_j}{h},0\right)$, respectively
$q = hg$	
r_o	ratio of slot width to distance between slot centers
v	upwash velocity
V	tunnel stream velocity
X,Y,Z	Cartesian coordinate axes
x,y,z	Cartesian coordinates
x_1,y_1	coordinates of image of a lifting element
x_1'	x-location variable of integration
α	a positive real parameter
$\Delta\alpha$	angle interference, deg

Γ circulation

$\delta(\omega)$ Dirac delta

δ upwash interference factor

δ_i upwash interference factor at point P_i

η,ξ coordinates of a lifting element

ρ density of test medium

φ velocity potential

φ_1 interference potential

φ_d velocity potential of a semi-infinite line doublet

φ_d' modified velocity potential defined by equation (3)

ω variable of transformation on $(x - x_1)$

Ω_d' exponential Fourier transform of φ_d' on $(x - x_1)$

Subscripts:

i position identification

j position identification

ANALYSIS

The lifting wing is represented by a distribution of semi-infinite doublet lines starting at discrete points of lift application on the sweptback wing and extending downstream toward infinity. For the justification of such representation see reference 4. The doublets are oriented as indicated in figure 1. The total interference is the sum of the interferences due to the interaction of the boundaries on the individual doublet lines. Figure 1 also shows the coordinate orientation relative to the test section of height h and width b. With the origin at the center of the test section, the coordinate x is in the

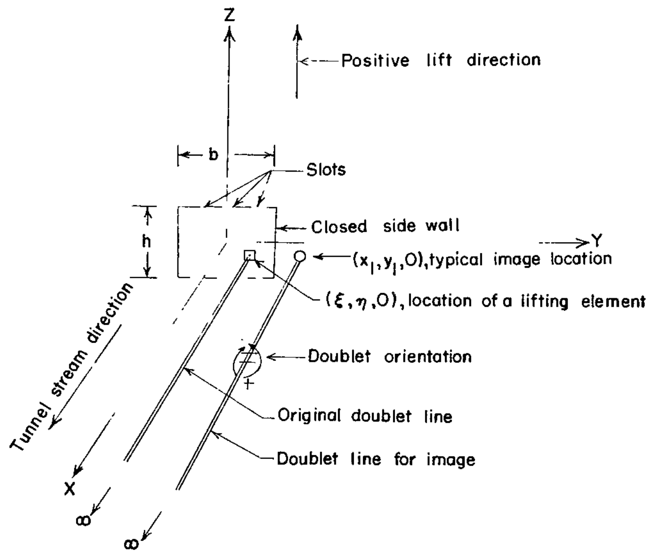

Figure 1.- Schematic drawing of test section and doublet configuration showing coordinate designation.

stream direction, y is normal to the closed side walls, and z is vertical, positive upward. Positive lift is taken in the positive z-direction. The boundary-induced upwash velocity, positive in the positive z-direction, due to a single lifting element located at some point $(\xi,\eta,0)$ in the horizontal center plane is now derived. The potential at a field point (x,y,z) due to a lifting semi-infinite line doublet starting at $\left(x_1,y_1,0\right)$ may be obtained by integrating the potential of a three-dimensional doublet over the line. The potential at a field point (x,y,z) of such a doublet located at $\left(x_1',y_1,0\right)$ is

$$d\varphi = \frac{\Gamma dA}{4\pi} \frac{z}{\left[\left(x - x_1'\right)^2 + \left(y - y_1\right)^2 - z^2\right]^{3/2}}$$

where ΓdA is the doublet strength, and Γ is the circulation enclosing the area dA of the elementary double source. (See ref. 5, page 203.) If a is the length of the span of the airfoil chosen for discrete representation by the doublet line, then $dA = a dx_1'$ and the potential due to the doublet line is

$$\varphi_d = \frac{\Gamma a}{4\pi} \int_{x_1}^{\infty} \frac{z dx_1'}{\left[\left(x - x_1'\right)^2 + \left(y - y_1\right)^2 + z^2\right]^{3/2}}$$

$$= \frac{\Gamma a}{4\pi} \frac{z}{\left(y - y_1\right)^2 + z^2}\left[1 + \frac{x - x_1}{\sqrt{\left(x - x_1\right)^2 + \left(y - y_1\right)^2 + z^2}}\right] \qquad (1)$$

Let

$$\varphi_d' = \frac{\Gamma a}{4\pi} \frac{z}{(y - y_1)^2 + z^2} \left[1 - \frac{1}{\alpha} \frac{\partial e^{-\alpha\sqrt{(x-x_1)^2+(y-y_1)^2+z^2}}}{\partial(x - x_1)} \right] \tag{2}$$

where $\alpha > 0$

Then

$$\varphi_d = \lim_{\alpha \to 0} \varphi_d' \tag{3}$$

By formula (26), p. 16, and formula (11), p. 118, of reference 6, and equation (3-6), p. 37, of reference 7, the exponential integral transform of φ_d' on $(x - x_1)$ with variable of transformation ω is

$$\Omega_d'(\omega,y,z) = \frac{\Gamma a}{2\pi} \frac{z}{(y - y_1)^2 + z^2} \left[\pi\delta(\omega) - \frac{i\omega\sqrt{(y - y_1)^2 + z^2}}{\sqrt{\omega^2 + \alpha^2}} K_1\left\{\sqrt{\left[(y - y_1)^2 + z^2\right](\omega^2 + \alpha^2)}\right\} \right] \tag{4}$$

for $\alpha > 0, \sqrt{(y - y_1)^2 + z^2} > 0$

where K_1 is the modified Bessel function of the second kind (called third kind in ref. 6). By formula (44), p. 56, and by formula (5), p. 118, of reference 6, the exponential integral transform of Ω_d' on $(y - y_1)$ with variable of transformation g is

$$G_d'(\omega,y,z) = \frac{\Gamma a}{2} \frac{z}{|z|} \left[\pi e^{-|z||g|}\delta(\omega) - \frac{i\omega e^{-|z|\sqrt{g^2+\omega^2+\alpha^2}}}{\omega^2 + \alpha^2} \right] \tag{5}$$

$$|z| > 0, \quad \sqrt{\omega^2 + \alpha^2} > 0, \quad \alpha > 0, \quad \sqrt{(y - y_1)^2 + z^2} > 0$$

With transformations on $(x - x_1)$ and $(y - y_1)$, the Laplace equation, $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$, transforms to

$$\frac{\partial^2 G}{\partial z^2} = (\omega^2 + g^2)G \tag{6}$$

A solution of this equation which has the required antisymmetry in z is

$$G_1(\omega,g,z) = A(\omega,g)\sinh\left(\sqrt{\omega^2 + g^2}z\right) \tag{7}$$

The equivalent homogeneous wall boundary condition (see ref. 2) is

$$\varphi \pm l\frac{\partial\varphi}{\partial z} = 0 \tag{8}$$

The positive and negative signs in equation (8) apply at the upper and lower boundaries, respectively, and l is the restriction constant given by equation (3) of reference 2 as

$$l = \frac{d}{\pi} \log_e \csc\left(\frac{\pi r_o}{2}\right)$$

where d is the distance between the centers of two adjacent slots and r_o is the ratio of slot width to the distance d, or for a uniformly slotted wall simply the proportion of the wall that is open. The restriction constant has the dimensions of a distance and approaches zero for an open tunnel and infinity for a closed tunnel. Equation (8) then gives the correct boundary conditions for these limiting cases and transforms to

$$G \pm l\frac{\partial G}{\partial z} = 0 \tag{9}$$

For positive z,

$$\frac{\partial G_d'}{\partial z} = \frac{\Gamma a}{2}\left[-\pi|g|e^{-z|g|}\delta(\omega) + i\omega\,\frac{\sqrt{g^2 + \omega^2 + \alpha^2}}{\omega^2 + \alpha^2}\,e^{-z\sqrt{g^2+\omega^2+\alpha^2}}\right] \qquad |z| > 0, \quad \alpha > 0 \tag{10}$$

and

$$\frac{\partial G_1}{\partial z} = A(\omega,g)\sqrt{\omega^2 + g^2}\cosh\left(\sqrt{\omega^2 + g^2}z\right) \tag{11}$$

Insertion of $\left(G_d + G_1\right)$ for G in the boundary condition (eq. (9)) for the upper boundary $z = \frac{h}{2}$ gives

$$\begin{aligned} &A(\omega,g)\left[\sinh\left(\sqrt{\omega^2 + g^2}\,\frac{h}{2}\right) + l\sqrt{\omega^2 + g^2}\cosh\left(\sqrt{\omega^2 + g^2}\,\frac{h}{2}\right)\right] \\ &= \lim_{\alpha\to 0} \frac{\Gamma a}{2}\left[\pi e^{-\frac{h}{2}|g|}\delta(\omega)\left(l|g| - 1\right) - \frac{i\omega e^{-\frac{h}{2}\sqrt{g^2+\omega^2+\alpha^2}}}{\omega^2 + \alpha^2}\left(l\sqrt{g^2 + \omega^2 + \alpha^2} - 1\right)\right] \end{aligned} \tag{12}$$

The same equation applies at the lower boundary, and it follows by solution for $A(\omega,g)$ and substitution into equation (7) that the transform of the interference potential is

$$G_1 = \lim_{\alpha\to 0} \frac{\Gamma a}{2} \frac{\left[\pi e^{-\frac{h}{2}|g|}\delta(\omega)\left(l\,|g| - 1\right) - \frac{i\omega e^{-\frac{h}{2}\sqrt{g^2+\omega^2+\alpha^2}}}{\omega^2+\alpha^2}\left(l\sqrt{g^2+\omega^2+\alpha^2} - 1\right)\right]\sinh\left(z\sqrt{\omega^2+g^2}\right)}{\sinh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right) + l\sqrt{\omega^2+g^2}\cosh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)} \qquad \alpha > 0 \tag{13}$$

The interference potential is thus

$$\varphi_1 = \lim_{\alpha\to 0} \frac{\Gamma a}{8\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\left[(x-x_1)\omega+(y-y_1)g\right]} \frac{\left[\pi e^{-\frac{h}{2}|g|}\delta(\omega)\left(l\,|g| - 1\right) - \frac{i\omega e^{-\frac{h}{2}\sqrt{g^2+\omega^2+\alpha^2}}}{\omega^2+\alpha^2}\left(l\sqrt{g^2+\omega^2+\alpha^2} - 1\right)\right]\sinh\left(z\sqrt{\omega^2+g^2}\right)d\omega\, dg}{\sinh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right) + l\sqrt{\omega^2+g^2}\cosh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)} \tag{14}$$

$$\alpha > 0$$

The upwash velocity in the XY-plane is

$$\left[\frac{\partial\varphi_1}{\partial z}\right]_{z=0} = \lim_{\alpha\to 0} \frac{\Gamma a}{8\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\left[(x-x_1)\omega+(y-y_1)g\right]} \frac{\left[\pi e^{-\frac{h}{2}|g|}\delta(\omega)\left(l\,|g| - 1\right) - \frac{i\omega e^{-\frac{h}{2}\sqrt{g^2+\omega^2+\alpha^2}}}{\omega^2+\alpha^2}\left(l\sqrt{g^2+\omega^2+\alpha^2} - 1\right)\right]d\omega\, dg}{\frac{\sinh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)}{\sqrt{\omega^2+g^2}} + l\cosh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)} \tag{15}$$

By taking the limit in equation (15), integrating the term containing $\delta(\omega)$, and noting the evenness in g, the upwash velocity v at $z = 0$ is

$$\left[\frac{\partial\varphi_1}{\partial z}\right]_{z=0} = \frac{\Gamma a}{8\pi^2} \int_{-\infty}^{\infty} \left\{ \frac{\pi e^{-\frac{h}{2}|g|}\left(l\,|g| - 1\right)}{\frac{\sinh\left(|g|\frac{h}{2}\right)}{|g|} + l\cosh\left(|g|\frac{h}{2}\right)} + \int_{-\infty}^{\infty} \frac{e^{-\frac{h}{2}\sqrt{g^2+\omega^2}}\left(l\sqrt{g^2+\omega^2} - 1\right)}{\frac{\sinh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)}{\sqrt{\omega^2+g^2}} + l\cosh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)} \frac{\sin\left[(x-x_1)\omega\right]}{\omega} d\omega \right\} \cos\left[(y-y_1)g\right] dg \tag{16}$$

The upwash interference factor is defined as

$$\delta = \frac{Cv}{AVC_L} \tag{17}$$

where

v	upwash velocity
V	tunnel stream velocity

C	cross-sectional area of test section
C_L	lift coefficient of model
A	area on which C_L is based

Moreover

$$\frac{C}{AVC_L}\frac{\Gamma a}{8\pi^2} = \frac{C}{16\pi^2}\frac{\rho V \Gamma a}{A\frac{1}{2}\rho V^2 C_L} = \frac{C}{16\pi^2}\frac{\Delta L}{L} \tag{18}$$

where ΔL is the element of lift represented by Γa and L is the total lift of the model. Also

$$C = hb$$

Then the contribution to the upwash factor at a point $(x,y,0)$ due to the slotted-wall influence associated with the lifting element image at $(x_1,y_1,0)$ is found from equations (16), (17), and (18) to be

$$\Delta\delta = \frac{hb}{8\pi^2}\frac{\Delta L}{L}\int_0^\infty\left\{\frac{\pi e^{-\frac{h}{2}g}(lg-1)}{\frac{\sinh\left(\frac{h}{2}g\right)}{g} + l\cosh\left(\frac{h}{2}g\right)} + 2\int_0^\infty \frac{e^{-\frac{h}{2}\sqrt{\omega^2+g^2}}\left(l\sqrt{\omega^2+g^2}-1\right)}{\frac{\sinh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)}{\sqrt{\omega^2+g^2}} + l\cosh\left(\frac{h}{2}\sqrt{\omega^2+g^2}\right)}\frac{\sin\left[(x-x_1)\omega\right]}{\omega}d\omega\right\}\cos\left[(y-y_1)g\right]dg$$

On changing the variables to $p = h\omega$ and $q = hg$,

$$\Delta\delta = \frac{1}{8\pi^2}\frac{b}{h}\frac{\Delta L}{L}\int_0^\infty\left\{\frac{\pi e^{-\frac{q}{2}}\left(\frac{l}{h}q-1\right)}{\frac{\sinh\frac{q}{2}}{q} + \frac{l}{h}\cosh\frac{q}{2}} + 2\int_0^\infty \frac{e^{-\frac{1}{2}\sqrt{p^2+q^2}}\left(\frac{l}{h}\sqrt{p^2+q^2}-1\right)}{\frac{\sinh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}{\sqrt{p^2+q^2}} + \frac{l}{h}\cosh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}\frac{\sin\left[\left(\frac{x}{h}-\frac{x_1}{h}\right)p\right]}{p}dp\right\}\cos\left[\left(\frac{y}{h}-\frac{y_1}{h}\right)q\right]dq \tag{19}$$

In equation (19)

$$\lim_{q\to 0}\frac{\sinh\frac{q}{2}}{q} = \lim_{\sqrt{p^2+q^2}\to 0}\frac{\sinh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}{\sqrt{p^2+q^2}} = \frac{1}{2}$$

and

$$\lim_{p \to 0} \frac{\sin\left[\left(\frac{x}{h} - \frac{x_1}{h}\right)p\right]}{p} = \frac{x}{h} - \frac{x_1}{h}$$

Let the original lifting element be located at $(\xi,\eta,0)$; then the element and its images in the solid side walls are all lifting elements and are located at $(\xi,kb - \eta)$ and at $(\xi,nb + \eta)$, where

$$k = \pm 1, \pm 3, \pm 5 \ . \ . \ .$$

and

$$n = 0, \pm 2, \pm 4, \pm 6 \ . \ . \ .$$

To every such lifting element there corresponds a contribution $\Delta\delta$ to the upwash interference factor arising from the top and bottom slotted walls and having the form of equation (19). A further contribution to the downwash interference factor is made by the images in the solid side walls. From equation (1) the upwash velocity produced at (x,y,z) by a lifting element image located at $\left(x_1,y_1,0\right)$ is

$$\frac{\partial\varphi_d}{\partial z} = \frac{\Gamma a}{4\pi}\left(\frac{(y - y_1)^2 - z^2}{\left[(y - y_1)^2 + z^2\right]^2} + \frac{(x - x_1)\left\{\left[(y - y_1)^2 + z^2\right]\left[(x - x_1)^2 + (y - y_1)^2 - 2z^2\right] - 2(x - x_1)^2 z^2\right\}}{\left[(y - y_1)^2 + z^2\right]^2\left[(x - x_1)^2 + (y - y_1)^2 + z^2\right]^{3/2}}\right)$$

By equations (17) and (18) the corresponding contribution to the upwash interference factor in the XY-plane, $z = 0$, is

$$\Delta\delta = \frac{hb}{8\pi}\frac{\Delta L}{L}\left[\frac{1}{(y - y_1)^2} + \frac{x - x_1}{(y - y_1)^2\sqrt{(x - x_1)^2 + (y - y_1)^2}}\right] = \frac{1}{8\pi}\frac{b}{h}\frac{\Delta L}{L}\frac{1}{\left(\frac{y}{h} - \frac{y_1}{h}\right)^2}\left[1 + \frac{\frac{x}{h} - \frac{x_1}{h}}{\sqrt{\left(\frac{x}{h} - \frac{x_1}{h}\right)^2 + \left(\frac{y}{h} - \frac{y_1}{h}\right)^2}}\right] \qquad (20)$$

Let $(\Delta\delta)_{ij}$ be the contribution to the upwash interference factor at a point $\left(x_i,y_i,0\right)$ corresponding to a lifting element located at a point $\left(\xi_j,\eta_j,0\right)$. Then by summing all the contributions of the form of equation (19) due to the top and bottom slotted walls and all contributions of the form of equation (20) due to the images in the side walls, the total contribution to the upwash interference factor at $\left(x_i,y_i,0\right)$ due to the lifting element at $\left(\xi_j,\eta_j,0\right)$ is found to be

$$
\begin{aligned}
(\Delta\,\delta)_{ij} = \frac{1}{8\pi^2}\,\frac{b}{h}\left(\frac{\Delta L}{L}\right)_j \Bigg[& \sum_k \left(\int_0^\infty \left\{ \frac{\pi e^{-\frac{q}{2}\left(\frac{\ell}{h}q - 1\right)}}{\dfrac{\sinh\frac{q}{2}}{q} + \dfrac{\ell}{h}\cosh\dfrac{q}{2}} + 2\int_0^\infty \frac{e^{-\frac{1}{2}\sqrt{p^2+q^2}\left(\frac{\ell}{h}\sqrt{p^2+q^2} - 1\right)}}{\dfrac{\sinh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}{\sqrt{p^2+q^2}} + \dfrac{\ell}{h}\cosh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}\;\frac{\sin\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)p\right]}{p}\,dp \right\} \cos\left[\left(\frac{y_i}{h} - k\frac{b}{h} + \frac{\eta_j}{h}\right)q\right] dq \right) \\
& + \sum_n \left(\int_0^\infty \left\{ \frac{\pi e^{-\frac{q}{2}\left(\frac{\ell}{h}q - 1\right)}}{\dfrac{\sinh\frac{q}{2}}{q} + \dfrac{\ell}{h}\cosh\dfrac{q}{2}} + 2\int_0^\infty \frac{e^{-\frac{1}{2}\sqrt{p^2+q^2}\left(\frac{\ell}{h}\sqrt{p^2+q^2} - 1\right)}}{\dfrac{\sinh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}{\sqrt{p^2+q^2}} + \dfrac{\ell}{h}\cosh\left(\frac{1}{2}\sqrt{p^2+q^2}\right)}\;\frac{\sin\left[\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)p\right]}{p}\,dp \right\} \cos\left[\left(\frac{y_i}{h} - n\frac{b}{h} - \frac{\eta_j}{h}\right)q\right] dq \right) \\
& + \sum_k \left\{ \frac{\pi}{\left(\frac{y_i}{h} - k\frac{b}{h} + \frac{\eta_j}{h}\right)^2} \left[1 + \frac{\frac{x_i}{h} - \frac{\xi_j}{h}}{\sqrt{\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - k\frac{b}{h} + \frac{\eta_j}{h}\right)^2}} \right] \right\} + \sum_{\substack{n \\ n\neq 0}} \left\{ \frac{\pi}{\frac{y_i}{h} - n\frac{b}{h} - \frac{\eta_j}{h}} \left[1 + \frac{\frac{x_i}{h} - \frac{\xi_j}{h}}{\sqrt{\left(\frac{x_i}{h} - \frac{\xi_j}{h}\right)^2 + \left(\frac{y_i}{h} - n\frac{b}{h} - \frac{\eta_j}{h}\right)^2}} \right] \right\} \Bigg]
\end{aligned}
\qquad (21)
$$

where

$$k = \pm 1, \pm 3, \pm 5 \ldots$$
$$n = 0, \pm 2, \pm 4, \pm 6 \ldots$$

The total upwash interference factor at $(x_i,y_i,0)$ due to all lifting elements in the presence of the tunnel boundaries is

$$\delta_i = \sum_j (\Delta\delta)_{ij} \tag{22}$$

SAMPLE CALCULATION

Let a wing swept back 35^o and spanning 0.7 of the test-section width be mounted at the center of a square, slotted wind-tunnel test section with solid side walls and with four symmetrically spaced slots in each of the other two walls as shown in figure 2.

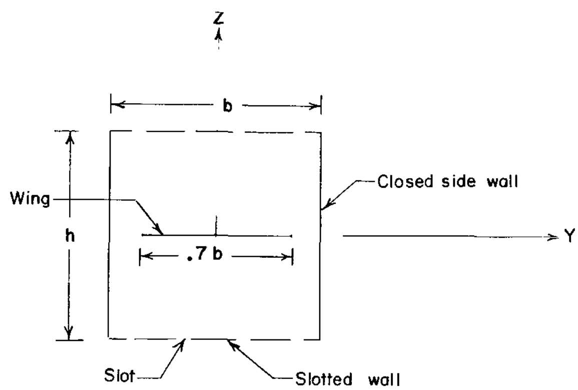

Figure 2.- Schematic drawing of cross section of wing and test section. For sample calculation sum of slot widths in boundary of width b is 0.06b.

Let the slots occupy 6 percent of the area of each slotted boundary. Then by application of equation (3) of reference 2 the restriction constant l is given by

$$\frac{l}{h} = \frac{d}{\pi h} \log_e \csc\left(\frac{\pi r_o}{2}\right) = \frac{\left(\frac{b}{4}\right)}{\pi h} \log_e \csc\left(\frac{\pi r_o}{2}\right)$$

$$= \frac{1}{4\pi}\left(\frac{b}{h}\right)\log_e \csc\left(\frac{\pi r_o}{2}\right)$$

$$= \frac{1}{4\pi} \log_e \csc\left(\frac{0.06\pi}{2}\right) = 0.18$$

Let the lifting wing be represented by lifting elements located at points $P_1, P_2, \ldots, P_{10}$ on lines of 35° sweep as shown in figure 3.

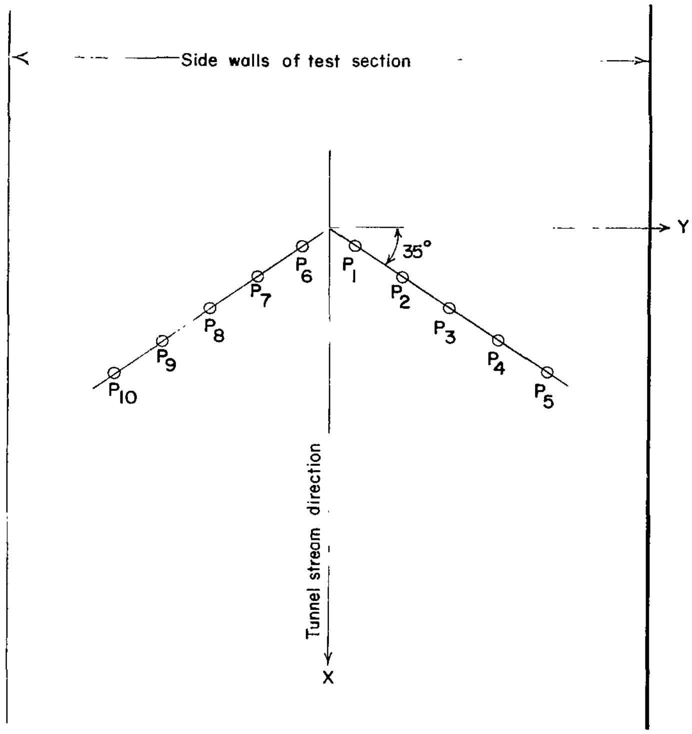

Figure 3.- Representation of sweptback wing for sample calculation.

The coordinates of these points and the lift distribution assumed are given in the following table:

Point	$\frac{\xi_j}{h}$ or $\frac{x_i}{h}$	$\frac{\eta_j}{h}$ or $\frac{y_i}{h}$	$\left(\frac{\Delta L}{L}\right)_j$
P_1	0.0246	0.0351	0.1342
P_2	.0738	.1054	.1334
P_3	.1229	.1756	.1118
P_4	.1721	.2458	.0769
P_5	.2212	.3160	.0437
P_6	.0246	-.0351	.1342
P_7	.0738	-.1054	.1334
P_8	.1229	-.1756	.1118
P_9	.1721	-.2458	.0769
P_{10}	.2212	-.3160	.0437

With substitution of $\frac{b}{h} = 1$ and $\frac{l}{h} = 0.18$ and with use of the coordinates $\frac{\xi_j}{h}$, $\frac{x_i}{h}$, $\frac{\eta_j}{h}$, and $\frac{y_i}{h}$ and of the lift distribution $\left(\frac{\Delta L}{L}\right)_j$ given in the table, the upwash interference factor $(\Delta\delta)_{ij}$ at any point P_i corresponding to a lifting element at any point P_j can be computed by use of equation (21), where the arguments of the sine and cosine functions must be carried to values large enough to assure convergence of the integrals with infinite upper limits and k and n must be carried to values large enough to assure convergence of the summations. The total upwash interference factor at point P_i is the sum of contributions from all points P_j and is given by equation (22) as

$$\delta_i = \sum_{j=1}^{10} (\Delta\delta)_{ij}$$

Since the wing is symmetrical about its midspan, it was necessary to compute δ_i for points on only one side of the midspan. The points $P_i = P_1, P_2, P_3, P_4,$ and P_5 were chosen. Computation of δ_i was made also for the points $P_0 = \left(\frac{x_i}{h}, \frac{y_i}{h}\right) = (0, 0)$ and $P_{11} = \left(\frac{x_i}{h}, \frac{y_i}{h}\right) = (0.2459, 0.350)$. (Note that P_0 and P_{11} are i-points, but not j-points, whereas all other points are both i-points and j-points.) The calculation was performed on an IBM 7094 electronic data processing system.

The calculated values of δ_i are shown in figure 4 as a function of spanwise position $\frac{2}{0.7}\frac{y_i}{h}$ on units of the semispan. For comparison the upwash-interference factor

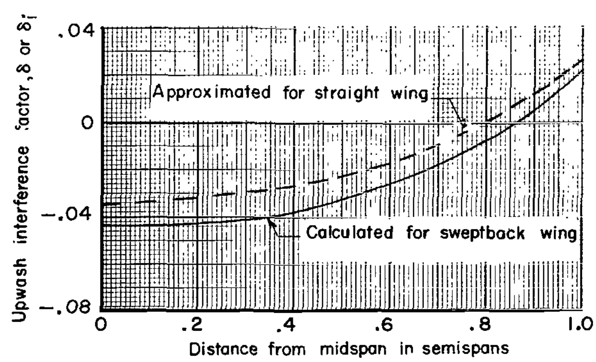

Figure 4.- Upwash interference factor for wing swept back 35° and spanning 0.7 of width of a square test section with four-slotted top and bottom walls and closed side walls, slot opening 6 percent of each slotted wall.

for a straight wing is also shown. The straight-wing interference was approximated by estimating the average upwash for an equivalent (0.9 span) uniformly loaded straight wing by use of reference 2 and adding the spanwise variation produced by the first five images in each side wall. The approximation is affected by the simplified loading assumed as well as by the simplified calculation procedure, but, within the accuracy of this approximation, it appears that the effect of sweep on the upwash velocities produced by the boundary interference is small for the example chosen.

For a lifting wing in a wind tunnel with slotted top and bottom walls and closed side walls, the slots permit a downflow relative to the upflow in a closed tunnel, and the slots do not have to be very wide for a net downflow to result, as shown by negative values of δ_i in the example. This effect is more clearly shown in reference 1. On the other hand, the closed side walls constrict the flow and produce a relative upflow which increases toward the walls. For the example shown the average interference factor is small, but the spanwise variation, which produces an effect like twist to increase the angle of attack toward the tip, may be significant. If the wing had spanned more than 0.7 of the width of the test section, this effect would have been greater. This effect increases ever more strongly as the wing tip approaches the wall.

To see what the interference factor of figure 4 means in terms of angle of attack on a practical model, suppose that the sample wing has an area of 0.06 of the cross-sectional area of the test section and is being tested at a lift coefficient of 0.6. Equation (17) gives for the angle interference in radians

$$\frac{v}{V} = \left(\frac{A}{C}\right) C_L \delta$$

so that in degrees the angle interference is

$$\Delta\alpha = 0.06 \times 0.6 \times 57.3 \times \delta$$

For the particular sweptback wing for which the values of δ_i are shown in figure 4, the corresponding values of the angle-of-attack increment $\Delta\alpha$ are presented in figure 5. The maximum tunnel-boundary interference on the angle of attack of the wing of this example is seen to be 0.09°, but the variation from root to tip is 0.14°.

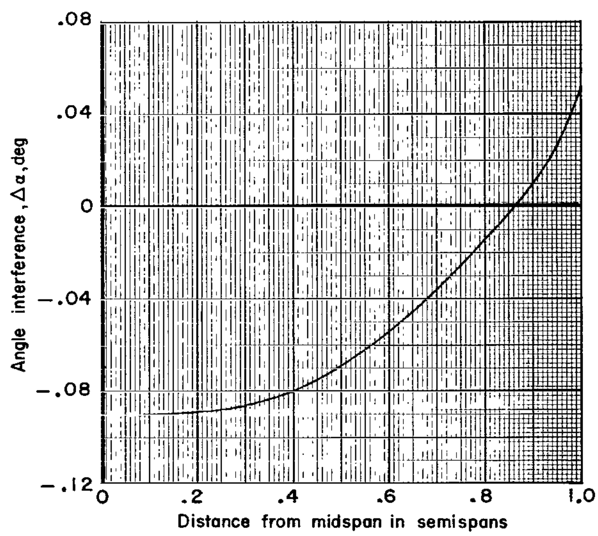

Figure 5.- Angle interference corresponding to δ for sweptback wing of figure 4. $A/C = 0.06$; $C_L = 0.6$.

DISCUSSION

The theory presented herein for the boundary-induced upwash interference due to lift on swept wings mounted at the centers of wind-tunnel test sections with slotted top and bottom walls and solid side walls is exact for the boundary conditions assumed. The accuracy of calculated results depends on accurate representation of the wing and on carrying the integrations and summations far enough to assure convergence. In consideration of the usual uncertainty in the knowledge of the lift distribution and of the approximate knowledge of the boundary conditions, it is believed that representation of the wing by 10 lifting elements as in the sample calculation is adequate. The use of homogeneous rather than exact slotted-wall boundary conditions is believed to introduce negligible error if there are several slots rather than only one or two in each slotted wall and if the wing does not approach a slotted wall. On the other hand, the action of the slots is uncertain and correspondingly so is the true effective restriction constant. The restriction constant used is calculated on the assumption of potential flow outward from the test section through slots with thin edges and no boundary layer. For outflow through coarse slots, this method of calculation should yield approximately correct values for the restriction constant, but for very narrow slots or for strong inflow from the plenum chamber surrounding the slots, the calculated values of the restriction constant may be appreciably in error. This latter situation may exist at the upper slotted wall for a model producing large lift values. The panels at the upper wall then become immersed in a thick boundary layer, so that they lose their effectiveness and the upper boundary condition may approach that for an open boundary. The effective restriction constant under such conditions needs experimental investigation.

Even if the effective restriction constant at the upper wall is appreciably different from the value assumed, it does not necessarily follow that the total boundary interference is greatly affected. This statement is supported by some unpublished results of research which show that the lift interference for a wing mounted at the center of a test section having solid side walls and one solid horizontal boundary with the other horizontal boundary open was almost the same as if all four boundaries had been closed. The boundary-induced upwash decreased strongly only as the wing was moved near to the open boundary. It seems reasonable to suppose that similar behavior would occur in the slotted tunnel as the lifting wing is moved nearer to the slotted upper wall. An experimental investigation of this effect would be desirable. If the slots are not of uniform width, an added uncertainty exists.

From these remarks and the results in the section entitled "Sample Calculation," it may be concluded that a lifting model tested in a wind tunnel with slotted top and bottom walls and solid side walls should be kept well away from the slotted walls and should span not more than 0.7 of the tunnel width. This statement suggests that the wing should be mounted at the center of the test section and that the tunnel height-to-width ratio should be not much less than 1. If the wing is sensitive to angle-of-attack variation over the span, a span of 0.7 of the tunnel width may already be too great. The comparison shown in figure 4 suggests that if the wing does not span more than 0.7 the tunnel width then the upwash interference can be well approximated without consideration of the wing sweep.

Within the applicability of linearized theory, the upwash interference factor is not affected by compressibility at subsonic speeds, provided the stream boundary, including the slots, runs approximately parallel to the tunnel stream direction. The theory presented herein may therefore be applied for subsonic compressible flow as well as for incompressible flow.

The theory can be used to calculate the upwash velocities anywhere in the XY-plane and thus corrections for the moment due to boundary-induced upwash at the tail or for the lift due to boundary-induced curvature of the flow can be applied. A compressibility correction can be made by applying the upwash velocities at $x\sqrt{1 - M^2}$ rather than at x, where x is the distance from the lifting element and M is Mach number. The flow curvature receives a compressibility factor $\frac{1}{\sqrt{1 - M^2}}$. For practical-size three-dimensional models, the lift correction due to flow curvature is commonly assumed to be negligible.

RÉSUMÉ

The upwash interference factor due to wind-tunnel boundary interference on the lift of sweptback wings center-mounted in test sections with slotted top and bottom walls and closed side walls has been obtained in the form of infinite convergent integrals and summations suitable for calculation by means of a high-speed digital computer. The interference factor is given as a function of position in the plane nominally containing the wing. In a sample calculation for a wing swept back 35° and spanning 0.7 of the test-section width the interference was found to be not much affected by the sweep. For this example the interference was everywhere small, but the solid side walls produced a sharp relative increase of the upwash toward the wing tips.

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REFERENCES

1. Matthews, Clarence W.: Theoretical Study of the Tunnel-Boundary Lift Interference Due to Slotted Walls in the Presence of the Trailing-Vortex System of a Lifting Model. NACA Rept. 1221, 1955. (Supersedes NACA RM L53A26.)

2. Davis, Don D., Jr.; and Moore, Dewey: Analytical Study of Blockage- and Lift-Interference Corrections for Slotted Tunnels Obtained by the Substitution of an Equivalent Homogeneous Boundary for the Discrete Slots. NACA RM L53E07b, 1953.

3. Katzoff, S.; and Hannah, Margery E.: Calculation of Tunnel-Induced Upwash Velocities for Swept and Yawed Wings. NACA TN 1748, 1948.

4. Von Kármán, Th.; and Burgers, J. M.: General Aerodynamic Theory – Perfect Fluids. Vol. II of Aerodynamic Theory, div. E, ch. III, sec. 12, W. F. Durand, ed., Julius Springer (Berlin), 1935 (reprinted by Durand Reprinting Comm., 1943), pp. 117-120.

5. Prandtl, L.; and Tietjens, O. G. (L. Rosenhead, trans.): Fundamentals of Hydro- and Aeromechanics. McGraw-Hill Book Co., Inc., 1934.

6. Staff of the Bateman Manuscript Project, compiler: Tables of Integral Transforms. Vol. I. McGraw-Hill Book Co., Inc., 1954.

7. Papoulis, Athanasios: The Fourier Integral and Its Applications. McGraw-Hill Book Co., Inc., 1962.

NASA-Langley, 1966 L-4812

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